The six equations formed from Eqs. (19-22) now yield the final result

$$\sigma_\theta(-) = \sqrt{s_{11}} = \sigma_n \sqrt{\left(\frac{x}{S_u}\right)^2 - 1} \qquad (30)\dagger$$

$$\sigma_\theta(+) = \sqrt{p_{11}} = \sigma_n \sqrt{1 - \left(\frac{S_u}{x}\right)^2} \qquad (31)$$

$$\sigma_w(-) = \sqrt{s_{22}} = \frac{\sigma_n}{T} \sqrt{S_u^2 \left(\frac{1}{x} + \frac{1}{2}\right) - x} \qquad (32)$$

$$\sigma_w(+) = \sqrt{p_{22}} = \frac{\sigma_n}{T} \sqrt{S_u^2 \left(\frac{1}{x} - \frac{1}{2}\right) - x} \qquad (33)$$

Problem 2

This system has no gyro, obtaining its attitude and attitude rate estimates by means of a dynamic model. The single-axis spacecraft motion is described by

$$\dot{\theta} = w \qquad (34)$$

$$\dot{w} = u \qquad (35)$$

where w is its angular velocity and u is the torque acting upon the spacecraft. Taking estimates of $\dot{\theta}$, $\dot{w}$, and u in the preceding equations, it follows that

$$\begin{bmatrix} \theta - \hat{\theta} \\ \hat{w} - w \end{bmatrix}_{t_{i+1}} = \Phi \begin{bmatrix} \theta - \hat{\theta} \\ \hat{w} - w \end{bmatrix}_{t_i} + \begin{bmatrix} f_{i+1} \\ g_{i+1} \end{bmatrix} \qquad (36)$$

which is the same as Eq. (7) except for the interchange in the positions of w and $\hat{w}$, and in which f_{i+1} and g_{i+1} are given by Eqs. (9) and (10) under the restrictions $n_v = 0$, $n_u = \hat{u} - u$. Subsequent analysis is now identical to problem 1, with Eqs. (28-33) again providing a recipe for obtaining steady-state performance. Here S_v is taken equal to zero, and S_u is given by Eq. (25), noting also Eq. (4). This completes the steady-state performance description of these two common spacecraft attitude estimators.

Example

A problem 1 type of attitude estimator updates its gyro every 600 s using a star tracker for which $\sigma_n = 20$ arc-s. The gyro, as a component, behaves such that, statistically, its drift rate changes 10^{-2} arc-s/s(1σ) in 12 h, whereas its integrated output changes 8.75 arc-s (1σ) in 0.5 h. What is the steady-state behavior of this estimator?

The Q matrix of Eq. (11) shows the gyro impact on attitude and drift rate for any time interval. σ_u is obtained from the drift rate specification, i.e.,

Source Eq.

$$\sigma_u^2(12 \times 3600) = (10^{-2})^2 \quad \therefore \quad \sigma_u = 4.81 \times 10^{-5} \text{ arc-s/s}^{3/2} \qquad (11)$$

$$\sigma_v^2(0.5 \times 3600) + \tfrac{1}{3}\sigma_u^2(0.5 \times 3600)^3 = (8.75)^2 \quad \therefore \quad \sigma_v = 0.200 \text{ arc-s/s}^{1/2} \qquad (11)$$

$$S_v = (0.2)(600)^{1/2}/20 = 0.245 \qquad (24)$$

$$S_u = (4.8 \times 10^{-5})(600)^{1.5}/20 = 0.035 \qquad (25)$$

$$\beta = 0.07122 \qquad (28)$$

$$x = -0.04234 \qquad (29)$$

†When $S_u = 0$, take $x/S_u = -\tfrac{1}{2}[S_v + \sqrt{S_v^2 + 4}]$.

The steady-state behavior now follows:

$$\sigma_\theta(-) = 13.2 \text{ arc-s} \qquad (30)$$

$$\sigma_\theta(+) = 11.0 \text{ arc-s} \qquad (31)$$

$$\sigma_w(-) = 0.0039 \text{ arc-s/s} \qquad (32)$$

$$\sigma_w(+) = 0.0037 \text{ arc-s/s} \qquad (33)$$

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Optimal Flare in Presence of Wind Shears

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Introduction

INVESTIGATIONS of several aircraft accidents indicate that wind shear is a factor that can cause aircraft crashes. The "Speckled Trout" C-135A advanced automatic landing flight-test program at the Air Force Flight Dynamics Laboratory included a hybrid simulation of the aircraft and automatic control system to investigate the effects of wind shear.[1] This study indicated that a logarithmically decreasing headwind from 50.67 ft/s (30 knots) at 510 ft AGL (above ground level) to 0 ft/s at 10 ft AGL resulted in an automatic landing 721 ft short of the no-wind case and a sink rate at touchdown of −6.2 ft/s instead of −2.1 ft/s for the no-wind case. From an altitude of 1000 to 510 ft, the wind was a constant 50.67 ft/s. This particular wind shear condition resulted in the worst performance compared to steady-state winds and linear shears of comparable magnitude. (Logarithmic winds were considered more representative of atmospheric conditions.) Considering the preceding logarithmic shear, a similar 30-knot headwind shear that is linear, and a constant 30-knot headwind, the longitudinal range dispersion at touchdown was 2017 ft, and the sink rate dispersion at touchdown was 5.55 ft/s. Dispersion is the span from the minimum to the maximum values; each extreme limit is determined by a wind condition. The longitudinal control equations were based on the conventional operational autopilot installed in the aircraft; during the time-based exponential flare, the throttles retard linearly.

The objective of this investigation is to design a flare control law that will enable an aircraft on automatic approach in the presence of unknown wind shears to achieve the smallest possible longitudinal dispersion and sink rate deviation at touchdown relative to a no-wind case. The flare portion of the automatic landing was chosen, since the flare control law is a major cause of poor landings in the presence of wind shears.[1]

The best possible performance should be obtained by using optimal control theory. Nonlinear equations of motion

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cannot be used, since a control law has to be obtained for every new set of initial conditions, and the computation speed is not great enough for real-time application. Linear equations of motion with a quadratic performance integral permit a solution that yields a control law independent of initial conditions; the optimal control gains are fixed prior to installation of an automatic landing system in an aircraft and are valid for any flare initial conditions.

Linear optimal control solutions for flare have been investigated. However, the effects of arbitrary wind shears on longitudinal dispersions and touchdown sink rates have not been addressed. Merriam[2] and Neal[3] did not consider wind effects. Applying their optimal control gains, which are functions of time, in a wind environment will not yield satisfactory results. Dispersions and sink rates will change considerably in the presence of winds, since the air mass is in motion. As an example, a constant headwind will cause the aircraft to be further from the no-wind touchdown point at a corresponding point in time. Huber[4] included a linear wind shear in his work, but the wind shear was known a priori. The wind shear was placed in the equations of motion; optimal control gains reflected a specific wind. The solution is valid for only that wind condition. Trankle[5] used an optimal regulator with integral control to track an altitude profile. This technique, however, can handle only constant winds. The results are not satisfactory if wind shears are introduced.

The approach in this investigation was to use linear optimal control theory and model the performance index after a pilot's performance. Specifically, a trajectory was not tracked. Instead, terminal weights were placed on pitch rate, airspeed, thrust, touchdown point, sink rate, and altitude. Since airspeed is critical during flare, an airspeed profile was tracked. Also, in order to keep normal acceleration small, in-flight weights were placed on pitch rate, normal airspeed, and elevator position. The controls were control column angular displacement and throttle angular displacement.

Furthermore, a crucial difference between this investigation and previous work that used time-varying optimal gains is the application of the time-varying gains based on range from a no-wind touchdown point. This scheduling is feasible, since range from a touchdown point is one of the aircraft states. Range may be determined in the future from a microwave landing system (MLS) or global positioning satellite system (GPS). As was mentioned previously, gain scheduling based on time in the presence of wind does not yield satisfactory results. In this study, time-varying gains are obtained for a no-wind condition. The gains are stored according to the range of the aircraft from the no-wind touchdown point. Constant wind and wind shears then are treated as disturbances to the system in simulations of the aircraft using the optimal no-wind control law. Also, in the simulations, ground effect is treated as a disturbance. But unlike wind, which cannot be predicted, a nominal ground effect model is included in the solution for the optimal gains.

System Model

Linearized longitudinal equations (stability axes) for perturbations about trimmed straight and level flight were used:

$$\ddot{\theta} = M_w w_{as} + M_{\dot{w}} \dot{w}_{as} + M_q \dot{\theta} + M_u u_{as} + M_{\delta T} \delta T + M_{\delta e} \delta e + M_H H$$

$$\dot{w} = Z_w w_{as} + Z_{\dot{w}} \dot{w}_{as} + (U_0 + Z_q)\dot{\theta} + z_u u_{as} + Z_{\delta e} \delta e + Z_H H$$

$$\dot{u} = -g\theta + X_w w_{as} + X_u u_{as} + X_{\delta T} \delta T + X_{\delta e} \delta e + X_H H$$

θ, w, and u are pitch angle, normal inertial velocity, and longitudinal inertial velocity, respectively, U_0 is steady-state airspeed, and g is acceleration due to gravity. The relationships between inertial and air mass velocities are

$$u = u_{as} + u_w; \quad w = w_{as}$$

where u_w is longitudinal wind velocity, and H is the ground effect parameter.

Change in elevator angle δe due to control column angular displacement δe_c is obtained through a first-order lag with 0.33-s time constant τ_e:

$$\dot{\delta e} = (-1/\tau_e)\delta e + (1/\tau_e)\delta e_c$$

There is also a first-order lag between throttle angular displacement δth_c and change in thrust δT with a 0.67-s time constant τ_{th}:

$$\dot{\delta T} = -(1/\tau_{th})\delta T + (750/\tau_{th})\delta th_c - 35313.8$$

The inertial position of the aircraft (center of gravity) is specified by its altitude above ground h and range r from a specified touchdown point on a runway:

$$\ddot{h} = U_0 \dot{\theta} - \dot{w}; \quad \dot{r} = U_0 + u$$

Using the previous equations, the full system is described by

$$\dot{x} = Ax + Bu + d_1 + d_2$$

$$x^T = [q \; \theta w_{as} \; u_{as} \; \delta T \; r \; \dot{h} \; h \; \delta e]$$

$$u^T = [\delta e_c \; \delta th_c]$$

where d_2 includes only wind terms.

Optimal Control Technique

The linear optimal control problem for the no-wind system with terminal and in-flight constraints may be described by the state equation

$$\dot{x} = Ax + Bu + d_1$$

and the performance index

$$J = \tfrac{1}{2}[x(t_f) - r(t_f)]^T S[x(t_f) - r(t_f)]$$

$$+ 1/2 \int_{t_0}^{t_f} \{[x(t) - r(t)]^T Q(t)[x(t) - r(t)]$$

$$+ u^T(t) R(t) u(t)\} dt$$

The desired terminal values $r(t_f)$ are

$$[q, u_{as}, \delta T, r, \dot{h}, h] = [0 \text{ rad/s}, -13.8 \text{ ft/s}, -23{,}540 \text{ lb}, 0 \text{ ft}, -2.5 \text{ ft/s}, 10 \text{ ft}]$$

The pitch rate constraint minimizes rotation at touchdown. The airspeed and sink rate specified are desired for the C-135 aircraft. The thrust constraint specifies idle thrust at touchdown. The range and altitude values identify a desired ground-referenced touchdown point. The in-flight constraints $r(t)$ are

$$[q, w, u_{as}, \delta e] = [0 \text{ rad/s}, 0 \text{ ft/s}, \; -0.191t^2 \text{ ft/s}, 0 \text{ rad}]$$

These constraints specify a trajectory with a smooth arresting of sink rate and a designated speed bleed profile.

The solution to a linear optimal control problem with terminal and in-flight constraints is given by Huber.[4] The control law is

$$u(t) = F(t)x(t) + v(t)$$

In determining F and v, ground effect was approximated by using the following altitude profile in order to specify d_1

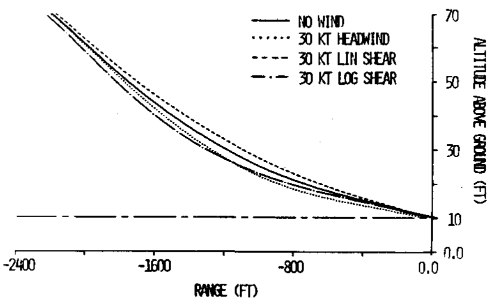

Fig. 1 Range trajectories with different initial conditions for various winds.

completely:

$$h = 70\exp(-0.255t)$$

It should be noted that d_2 was not included in the computation.

With the optimal control law specified, the no-wind state equation was integrated by implementing a fourth-order Runge-Kutta algorithm. The weights that resulted in the best adherence to the in-flight and terminal constraints are

$$S = \text{diag}[10^8 \;\; 0 \;\; 0 \;\; 10^6 \;\; 10^2 \;\; 10^2 \;\; 10^6 \;\; 10^6 \;\; 0]$$

$$Q = \text{diag}[10^4 \;\; 0 \;\; 10^4 \;\; 10^6 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 10^4]$$

$$R = \text{diag}[5\times 10^6 \;\; 10^3]$$

Based on the no-wind optimal solution, $F(t)$ and $v(t)$ were tabulated with respect to the range associated with each time interval, $F(r)$ and $v(r)$. Simulation of flare in wind conditions then was accomplished with the following equations:

$$\dot{x} = Ax + Bu + d_1 + d_2$$

$$u = F(r)x + v(r)$$

where r is the state specified in the state vector x. The disturbance vector d_1 includes ground effect, which is a function of altitude. The wind disturbance vector d_2 also is specified as a function of altitude.

It should be noted that the in-flight and terminal constraints, along with their weights, were not fully determined prior to simulation with winds. Simulation in wind conditions aided in identifying the constraints and weights that were selected.

Results

The optimal flare was simulated on a CDC 6600 digital computer. Initial conditions for the flare were obtained from the "Speckled Trout" C-135 hybrid simulation mentioned in the Introduction; the state vectors at flare initiation for the no-wind, 30-knot constant headwind, 30-knot linear headwind shear, and 30-knot logarithmic headwind shear were used. Stability derivatives were for a C-135A aircraft in landing configuration with a steady-state airspeed of 261.8 ft/s and gross weight of 160,000 lb. Results are shown in Fig. 1. Full analysis of the state and control vectors is presented in Ref. 6. The simulation indicated that the optimal control gains based on range provide smooth flare trajectories with a longitudinal touchdown dispersion of only 71.1 ft and a sink rate dispersion at touchdown of only 0.17 ft/s. As mentioned previously, the "Speckled Trout" C-135 simulation yielded range and sink rate dispersions of 2017 ft and 5.55 ft/s. Thus, compared to a conventional operational autopilot, the linear optimal control law permits a very significant reduction in dispersions.

It should be noted that the optimal control technique presented here is good for placing an air vehicle at any point in space, with any terminal conditions, with any in-flight constraints, in any deterministic wind conditions; this can be done regardless of the vehicle's initial conditions. Further investigation should analyze the gust response of the system.

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Nonlinear Method for Parameter Identification Applied to a Trajectory Estimation Problem

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Introduction

A PRIMARY objective of vehicle flight testing is to determine the aerodynamic characteristics of the vehicle. In some cases, the characteristic being sought is a parameter – for example, a stability derivative at a given Mach number. In other cases, it can be a function of one or more variables – for example, the nose tip shape of an ablating re-entry vehicle. The identification of a function can be reduced to the identification of parameters by letting the parameters be values of the function at a number of points and using curve fitting to form the function.

The parameters are identified by minimizing the least-squares performance index, using a variable-metric optimization algorithm and numerical partial derivatives. Hence, only the model defining the motion and initial guesses of the parameters are needed to do the parameter estimation. This procedure has been developed because it eliminates the setup time involved with using the linearized equations of motion characteristic of most estimation algorithms. Because of this feature, this approach can be useful for experimenting with modeling.

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